chemical kinetics transport in the stratosphere, while Loeb and Schiesser study how the eigenvalues and stiffness ratio vary with the number of points in the spatial difference grid for a model problem.

Asymptotic approximation appears in several of the articles. Lapidus, Aiken and Liu survey the occurrence of stiff physical and chemical systems and show certain relationships among pseudo-steady-state approximation, singular perturbations, and stiff systems. Kreiss' paper deals with solutions for singular perturbations of two-point boundary value problems.

Stetter analyses and extends some novel ideas of Zadunaisky on global error estimation using nonstandard local error estimates. Hachtel and Mark combine polynomial prediction and truncation error control with a Davidenko-type parameter stepping method for nonlinear algebraic equations. Bulirsch and Branca briefly discuss computation in real-time-control situations.

As one can see from the topics mentioned here, this book is not for the novice. However, for the mature reader, it is an excellent guide to the literature on and introduction to the many difficult aspects of stiff equations.

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50 [9.00, 9.20].—SAMUEL YATES, *Prime Period Lengths,* 104 Brentwood Drive, Mt. Laurel, N. J., 1975, ii + 131 pp. Price \$10.00 (paperbound).

This is a privately printed and bound version of the author's UMT previously reviewed in [1], which one should see for additional description. The new version achieves a reduction in size by a factor of 8 by printing four reduced-size pages of the previous table on each side of a page. The main content, as before, is a list of the 105000 primes $p \leqslant 1370471$ (excluding $p = 2$ and 5) versus the period P of the decimal expansion of the reciprocal $1/p$. Of course, P is also the order of $10 \pmod p$ and what the author calls "full-period primes" (those with $P = p - 1$) are the primes having 10 as a primitive root.

The preface indicates that the main purpose of this publication is to enable investigators to study questions of distribution and to formulate appropriate conjectures. That being the case, it is surprising that the author does not include derived tables of such distributions, e.g., a table of the distribution of the "full-period primes." The reviewer agrees that there are interesting distribution problems here. He must admit, though, that he has also used the table for a much simpler purpose, namely, as a convenient list of primes $\leqslant 1370471$.

The one-page introduction contains a passage of such ambiguity that it must be quoted in full:

"Asymptotically speaking,

a. The period lengths of all primes are distributed evenly among the sixteen possible residue classes (mod 40).

b. The period lengths of half of all the primes congruent to 13 or 37 (mod 40) are even, and half are odd.

c. The period lengths of five sixths of all primes congruent to 1 or 9 (mod 40) are even, and one sixth are odd.

d. The period lengths of two thirds of all primes are even, and one third are odd.

e. If we divide all primes into three categories—full-period, odd period length, and non-full-period with even period length—the ratio of totals to each other in the given order, is 9:8:7.

These assertions are among others in articles by Samuel Yates and by Daniel Shanks in The Journal of Recreational Mathematics, beginning in 1969."

Since the five propositions are all called "assertions" it is not clear here which are true and which are false, and since no more exact references are given the reader would have a problem in determining which of the five "assertions" are due to which of the two named authors.

The facts are these: Proposition (a) is simply a special case of de la Vallée Poussin's famous theorem (1896) concerning primes in an arithmetic progression [2]. Proposition (d) was conjectured by Krishnamurthy [3] and proven by me in [4]. In the proof, (b) and (c) are preliminary results. Assertion (e), I am happy to report, is solely due to Yates. There is no reason to think that it is true and much reason to think that it is false.

If $\nu_{10}(x)$ is the number of primes $\leqslant x$ having 10 as a primitive root, then Assertion (e) would follow from proposition (d) if, and only if,

$$r_{10}(x) = \nu_{10}(x)/\pi(x) \longrightarrow 3/8 = 0.375 \tag{1}$$

as $x \longrightarrow \infty$. But there is every reason to believe that (1) is false. Heuristically, the correct asymptote is almost certainly the somewhat smaller Artin's constant:

$$A = \prod_{p=2}^{\infty} \left(1 - \frac{1}{p(p-1)}\right) = 0.3739558, \tag{2}$$

and Hooley [5] has proven this by assuming certain Riemann hypotheses.

The *empirical* case for (1) might seem more favorable at first. Up to Yates' limit $x = 1370471$, $r_{10}(x)$ is usually $> A$ (see [1] for an exception), and $r_{10}(x)$ is even $> 3/8$ throughout much of this range. The closing quotation is $r_{10}(1370471) = 0.37568$. However, it has long been known [6] that the convergence to A should be mostly from above. The first factor in (2), from $p = 2$, equals 1/2 and represents the fraction of primes having 10 as a quadratic nonresidue. While 1/2 is the correct asymptotic proportion of such primes, for finite x the fraction is usually [7] slightly more than 1/2 and, therefore, $r_{10}(x)$ will usually exceed A. In any case, in the following UMT, by Baillie [8], $\nu_{10}(x)$ (and other $\nu_a(x)$) are extended out to $x = 33 \cdot 10^6$. One finds this: after $x = 2.1 \cdot 10^6$, $r_{10}(x)$ remains < 0.37500; after $3.2 \cdot 10^6$, $r_{10}(x) < 0.37475$; after $9.8 \cdot 10^6$, $r_{10}(x) < 0.37450$ and after $14.1 \cdot 10^6$, $r_{10}(x) < 0.37425$.

While that proves nothing about $r_{10}(x) \longrightarrow A$, it does show that there is no case whatsoever for (1) and therefore no case whatsoever for Assertion (e). The correct proportions are almost surely not 9:8:7 but rather the less elegant 8.97494:8:7.02506.

D. S.

1. SAMUEL YATES, "Prime period lengths," UMT 10, *Math. Comp.*, v. 27, 1973, p. 216.

2. Ch. de la VALLÉE POUSSIN, "Recherches analytiques sur la théorie des nombres premiers, deuxième partie," *Ann. Soc. Sci. Bruxelles*, v. 20, part 2, 1896, pp. 281–362.

3. E. V. KRISHNAMURTHY, "An observation concerning the decimal periods of prime reciprocals," *J. Recreational Math.*, v. 4, 1969, pp. 212–213.

4. DANIEL SHANKS, "Proof of Krishnamurthy's conjecture," *J. Recreational Math.*, v. 6, 1973, pp. 78–79.

5. CHRISTOPHER HOOLEY, "On Artin's conjecture," *Crelle's J.*, v. 225, 1967, pp. 209–220.

6. DANIEL SHANKS, *Solved and Unsolved Problems in Number Theory*, Spartan, Washington, D.C., 1962, p. 83.

7. DANIEL SHANKS, "Quadratic residues and the distribution of primes," *MTAC*, v. 13, 1959, pp. 272–284. See Table 6.

8. ROBERT BAILLIE, *Data on Artin's Conjecture*, UMT 51, *Math. Comp.*, v. 29, 1975, pp. 1164–1165.